LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2025**

UMT 4501 – REAL ANALYSIS-I

Date: 24-04-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions - **(10 x 1 = 10)**
1.	**Fill in the blanks**
a)	If $f(x) = x^2, g(x) = x$, then $(g \circ f)(x) =$ ________.
b)	There does not exist a rational number r such that ________.
c)	Every nonempty set of real numbers that has an upper bound also has a ________ in R.
d)	A convergence sequence of real number is __________.
e)	The limit of the sequence $1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}-\ln n$ is called _________.
2.	**MCQ**
a)	If A is a set with m elements and B is a set with n elements and if $A \cap B = \phi$ then $A \cup B$ has ---- elements. (i) m^2+n^2 (ii) $m-n$ (iii) $m+n$ (iv) m^2
b)	If $a,b \in R$ then $\|\|a\|-\|b\|\| \leq$ (i) $\|a+b\|$ (ii) $\|a-b\|$ (iii) $\|ab\|$ (iv) $\|a+2b\|$
c)	If $S = \left\{\frac{1}{n} : n \in N\right\}$ then (i) $\sup S = 1$ (ii) $\inf S = 0$ (iii) $\inf S = 1$ (iv) $\sup S = 0$
d)	If $0 < b < 1$, then $\lim(b^n) =$ (i) 1 (ii) 0 (iii) ∞ (iv) -1
e)	$\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent when (i) $p=1$ (ii) $p<1$ (iii) $p>1$ (iv) $0 \leq p \leq 2$
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**True or False**
a)	The set of natural numbers is a finite set.
b)	If $a,b \in R$ then $\|a+b\| \geq \|a\|+\|b\|$.
c)	For all $a,b \in R$, $\|ab\| = \|a\|\|b\|$.
d)	$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ..., \frac{1}{n}$ is an increasing sequence.
e)	$1+r+r^2+...+r^n+...$ is a geometric series.
4.	**Answer the following**
a)	State the principle of strong induction.

b)	Write down Bernoulli's inequality.
c)	State Cantor's Theorem.
d)	Define sequence of real numbers.
e)	Define limit of a sequence.
SECTION B - K3 (CO2)	
Answer any TWO of the following	**(2 x 10 = 20)**
5.	Using the principle of mathematical induction, demonstrate that $1^3+2^3+3^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$.
6.	Establish the characterization Theorem of intervals with an appropriate proof.
7.	Produce the proof for monotone convergence Theorem.
8.	State Raabe's Test and hence test the convergence of the series $\sum_{n=1}^{\infty}\frac{2\cdot4\cdot6\cdot\ldots\cdot2n}{1\cdot3\cdot5\cdot\ldots\cdot(2n+1)}$ using it.
SECTION C – K4 (CO3)	
Answer any TWO of the following	**(2 x 10 = 20)**
9.	If S and T are sets and that $T\subseteq S$, T is an infinite set, then show that S is an infinite set.
10.	Establish nested interval property with a suitable proof.
11.	Prove that the interval $[0,1]=\{x\in R:0\le x\le 1\}$ is not countable.
12.	Categorise the convergence of the series whose nth term is $\frac{1}{(n+1)(n+2)}$.
SECTION D – K5 (CO4)	
Answer any ONE of the following	**(1 x 20 = 20)**
13.	(a) Establish that countable union of countable sets is countable. (b) State and establish the density theorem. (10+10)
14.	(a) State and prove Bolzano-Weierstrass Theorem. (b) Test the convergence of the series $\sum a_n cosnx$ by Dirichlet's test. (12+8)
SECTION E – K6 (CO5)	
Answer any ONE of the following	**(1 x 20 = 20)**
15.	(a) Prove that the set of all rational numbers is denumerable. (b) Develop the relation between Arithmetic mean and Geometric mean in the form of inequality and prove it. (8+12)
16.	(a) Establish Archimedean property with an appropriate proof. (b) Show that $\lim\left(\frac{3n+2}{n+1}\right)=3$. (c) Show that $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$ converges by Cauchy's integral test. (6+6+12)

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